national accelerator laboratory

Semi-Ellipses and The Definition of Horizontal

Phase Spaces In The Transfer Hall

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It has become customary with a number of us to define and measure

the phase space area of the extracted proton beam in the Transfer Hall,

and at points further downstream, in the following manner: the full

width at half maximum (FWHM) of a number of SWIC histograms is measured

and is then taken to be the half width of the projection of an elliptical

phase space, inaccurately called the "two-standard-deviation phase space".

The ellipse parameters and the area are then deduced from a least-squares

fit to the widths of four to six SWIC images. The result has been horizontal

phase space areas in the range (0.2 - 0.3) π mm-mrad, depending upon

which SWICs are used. From measuring the fraction of the particles

falling outside of 2 SD on the SWIC histograms, I conclude that this

phase space includes $\sim$ 85% of the particles.

On the other hand, phase spaces just downstream of ES40, the

extraction electrostatic septum, are measured and defined quite differently.

From target scans and a little bit of theory, the shape is believed to

be either rectangular or half-elliptical, as shown in Fig. 1. The area

has been estimated to be 0.044 π mm-mrad at 300 GeV by Fisk, and 0.067

π mm-mrad by H. Edwards at 200 GeV, and includes 100% of the unscattered

particles. The shape of right hand side of Fig. 1 is unknown.

A question which needs answering is, how much of this alarming

growth of the horizontal emittance is real and how much is the result

of the inconsistent definitions of phase space? Some people have suggested

that the growth could be entirely due to inconsistent definitions.

This cannot be the case, since the time-dependence of the position of

the spot on flag MW100 (it oscillates, probably 15 cps) has been estimated

to double the effective spot size on that flag (eyeball estimate).

In an attempt to gain a feel for the definition problem, I have

answered the following very limited and somewhat hypothetical question:

if the phase space at ES40 is a uniformly populated half-ellipse of area

0.044 π mm-mrad, what would the SWIC histograms look like in the Transfer

Hall and what would be the result of a least-squares fit to their full

widths at half maximum, as described in the first paragraph?

Table I lists the FWHM of the hypothetical half-ellipse at the

six switchyard SWICs (MQ90-91 off) and the observations of Fisk and

Hornstra in July, at 300 GeV. The least squares fit to the half-ellipse

widths yields an emittance of 0.105 π mm-mrad, i.e., the growth factor

is about 2.4. The predicted SWIC distributions are shown in Figs. 2a

and 2b, and can be compared to the observed SWIC distributions of Figs.

3a and 3b (July 1973 data). The orientation of the half-ellipse at PSEP

is shown in Fig. 1b. The last two columns of Table I show that the

widths of the images of the half-ellipse model, plus the effect of an

assumed ± 0.125% ripple in the extraction bends, are in modest agreement

with the observations. Sho Ohnuma demonstrated the same fact, using

a full-ellipse model.

Three other different half-ellipses have been traced through

the program, with roughly the same "growth factor" (2.1 to 2.5). Others

can be traced most easily, if anyone is interested.

TABLE I

FULL WIDTH AT HALF MAX., HORIZONTAL PLANE		DISPERSION DUE TO EXTRACTION CHANNEL	HALF-ELLIPSE FWHM PLU 0.125% DISPERSION
SWIC	OBSERVED	HALF-ELLIPSE MODEL		
AO	3.35 mm	2.89 mm	0.02 mm/0.1%	2.92 mm
VH	?	2.69	0.07	2.78
MQ90	3.95	2.52	0.82	3.52
PSEP	4.25	3.17	1.26	4.75
MH300	6.50	4.48	2.39	7.48
MQ100	11.90	6.49	3.25	10.55

300 GeV phase space (horizontal) at ES4O

X' μrad

X mm

300 GeV phase space (horizontal) transformed to PSEP (MQ90-91m)

X' μrad

X mm

Fig. 1

N(x)

SWIC AD

X (mm)

SWIC VH

X (mm)

SWIC MQ9D

X (mm)

X (mm)
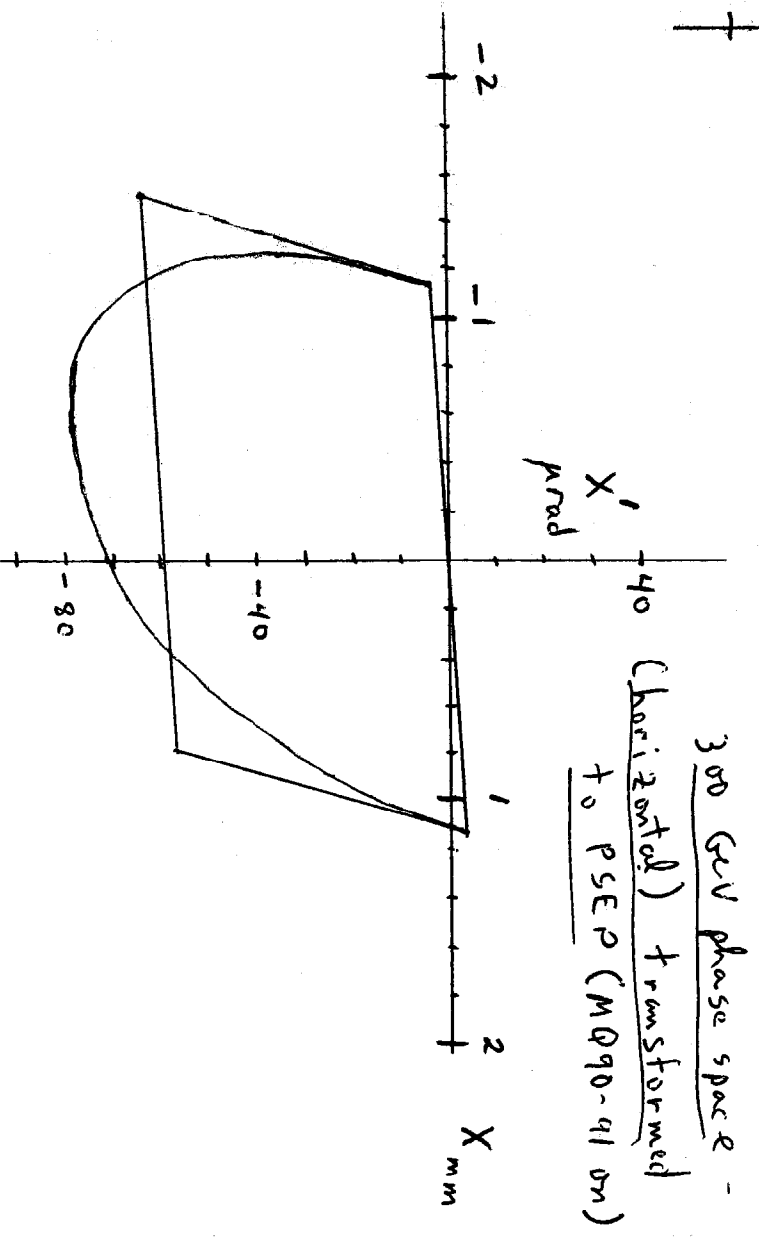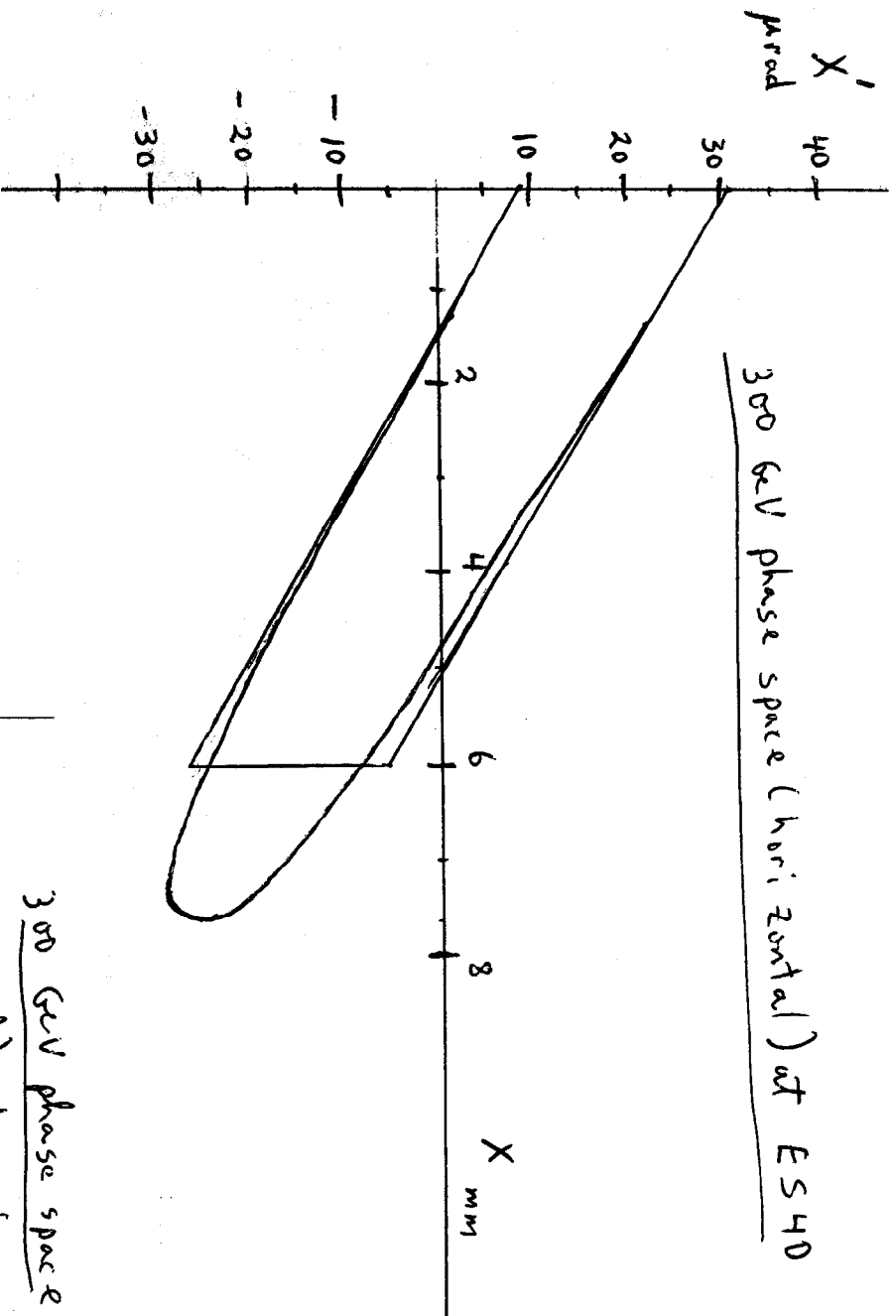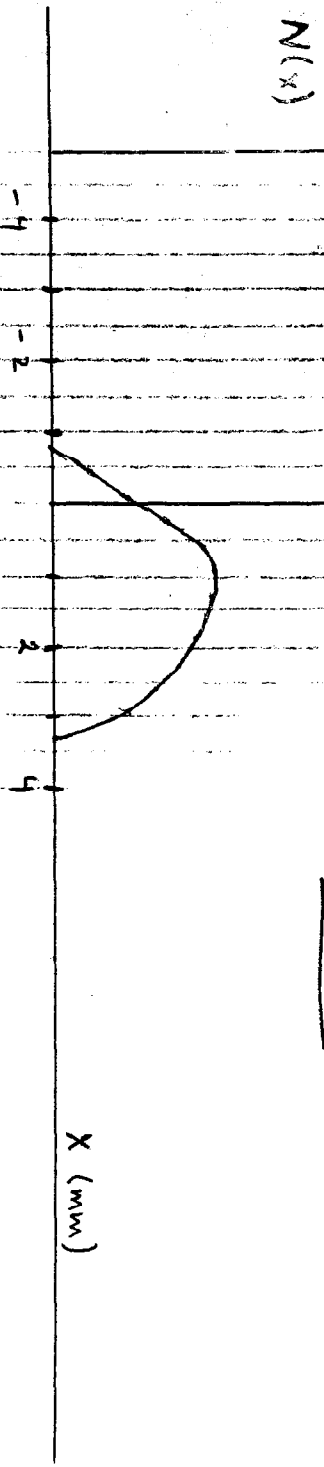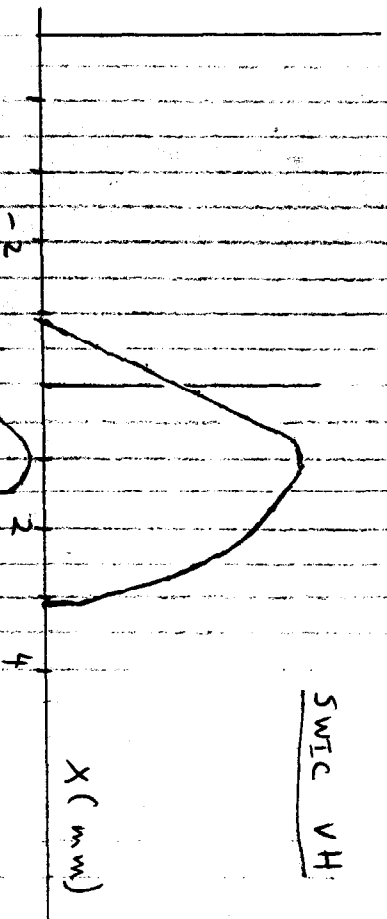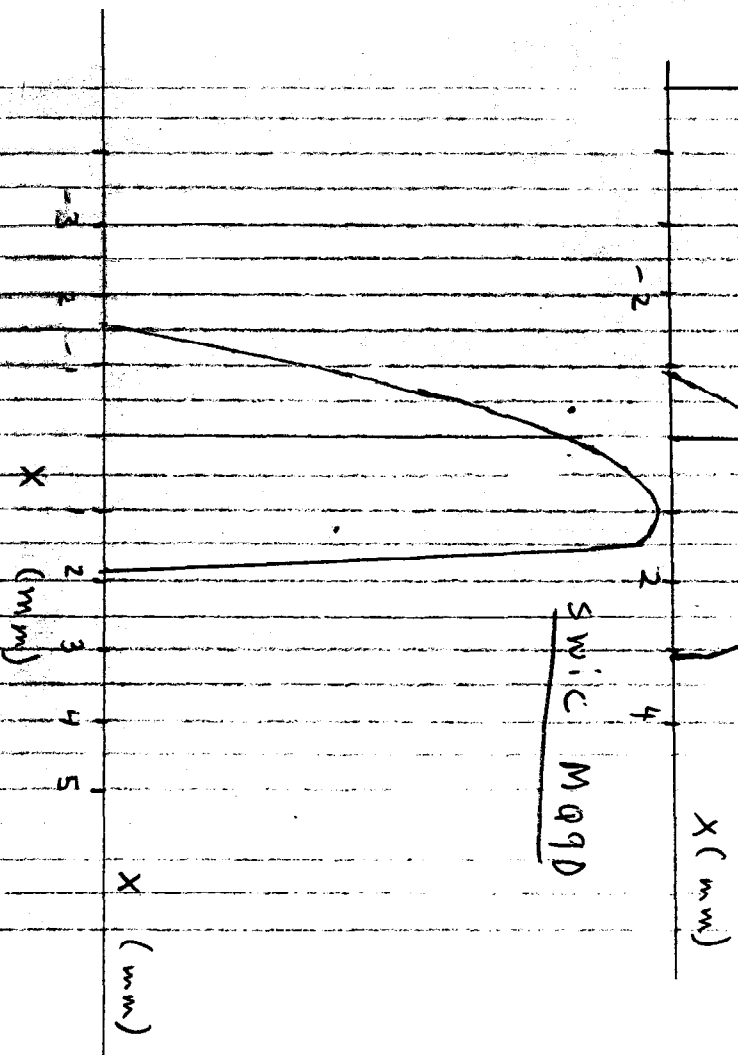
Fig. 2a

Projections of the transformation of the half-
ellipse of Fig. 1

Fig. 2(b)

MQ100

MH300

PSEP

X (mm)

X (mm)

X (mm)

6

TM-454
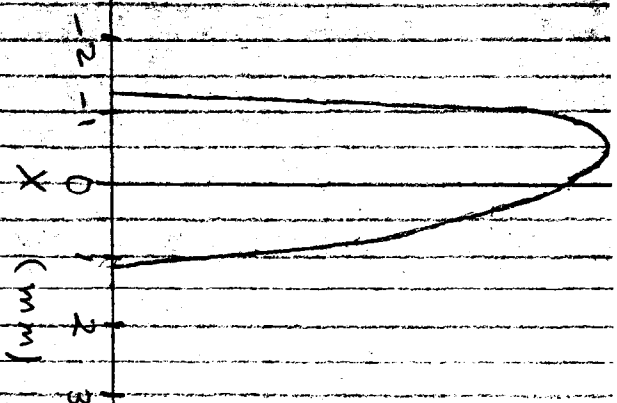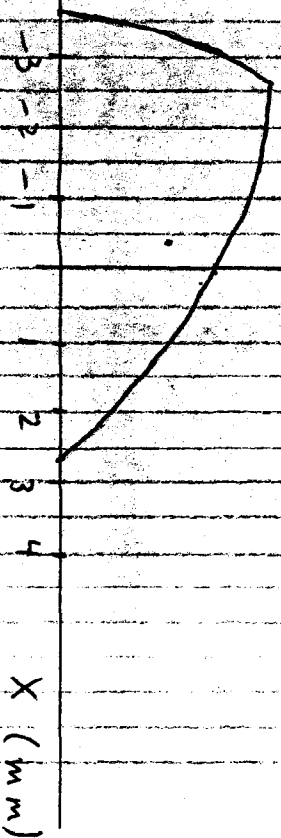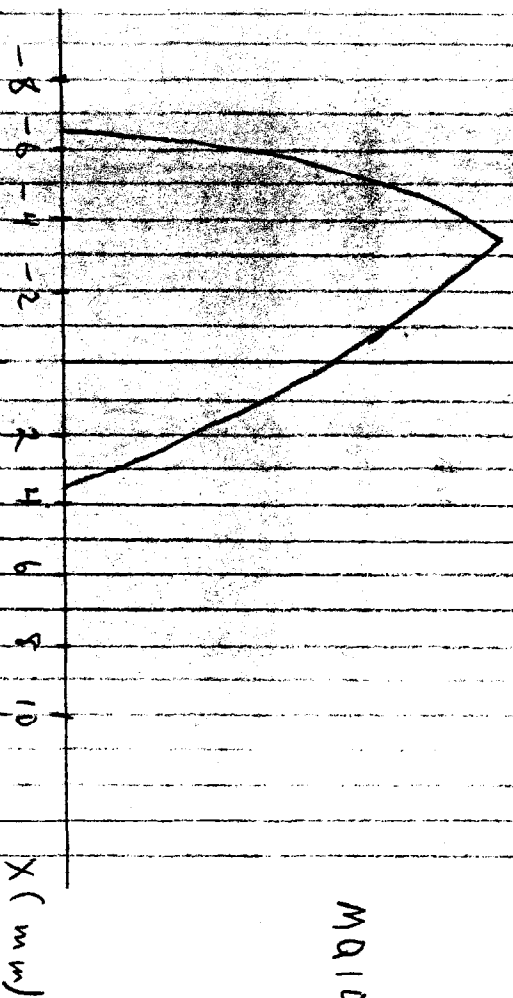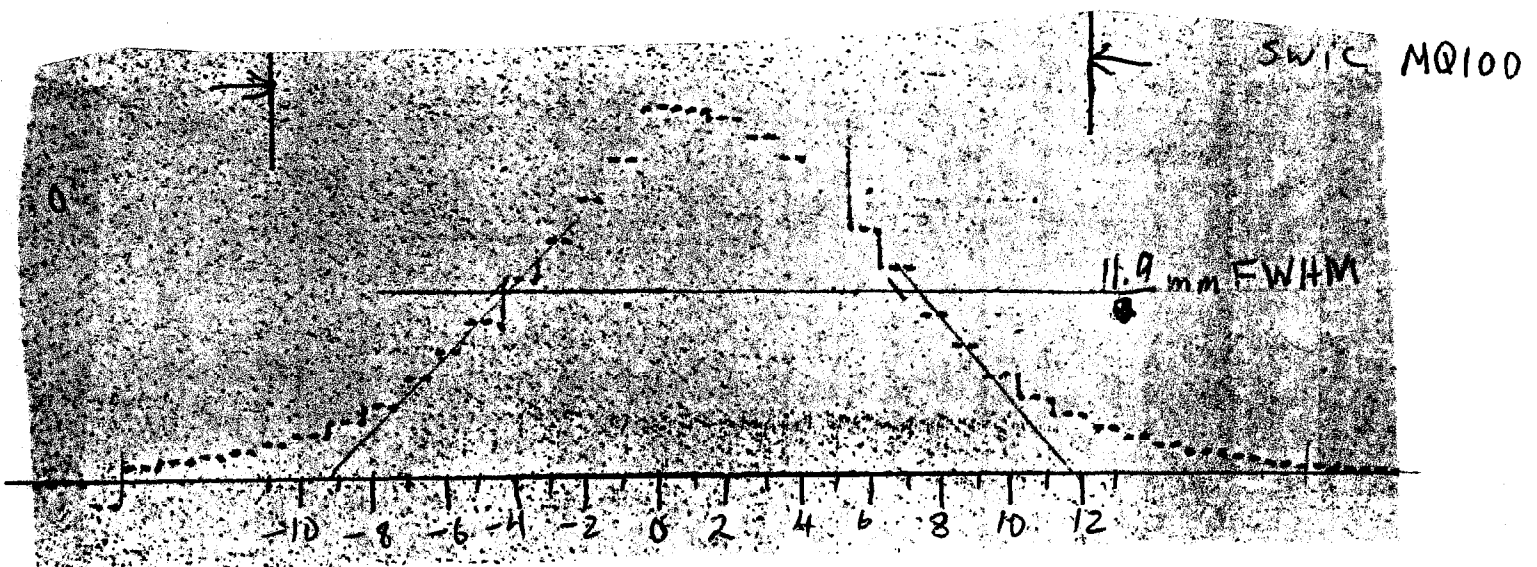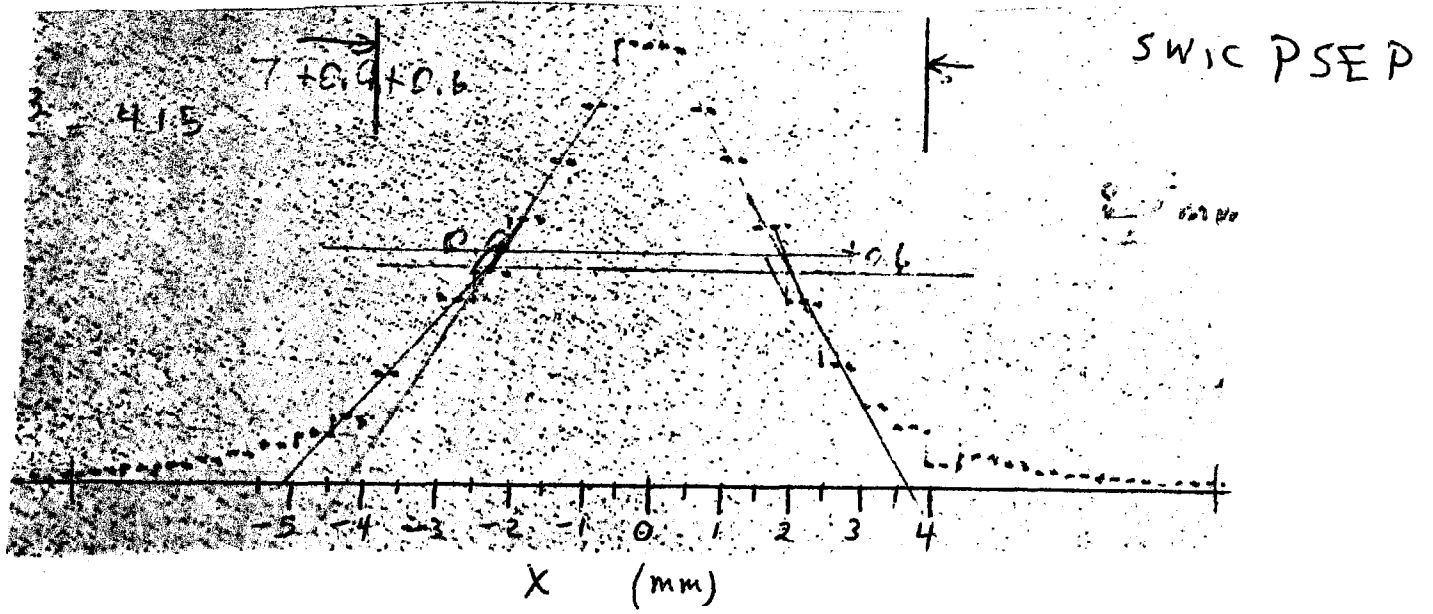
Fig. 3(b)
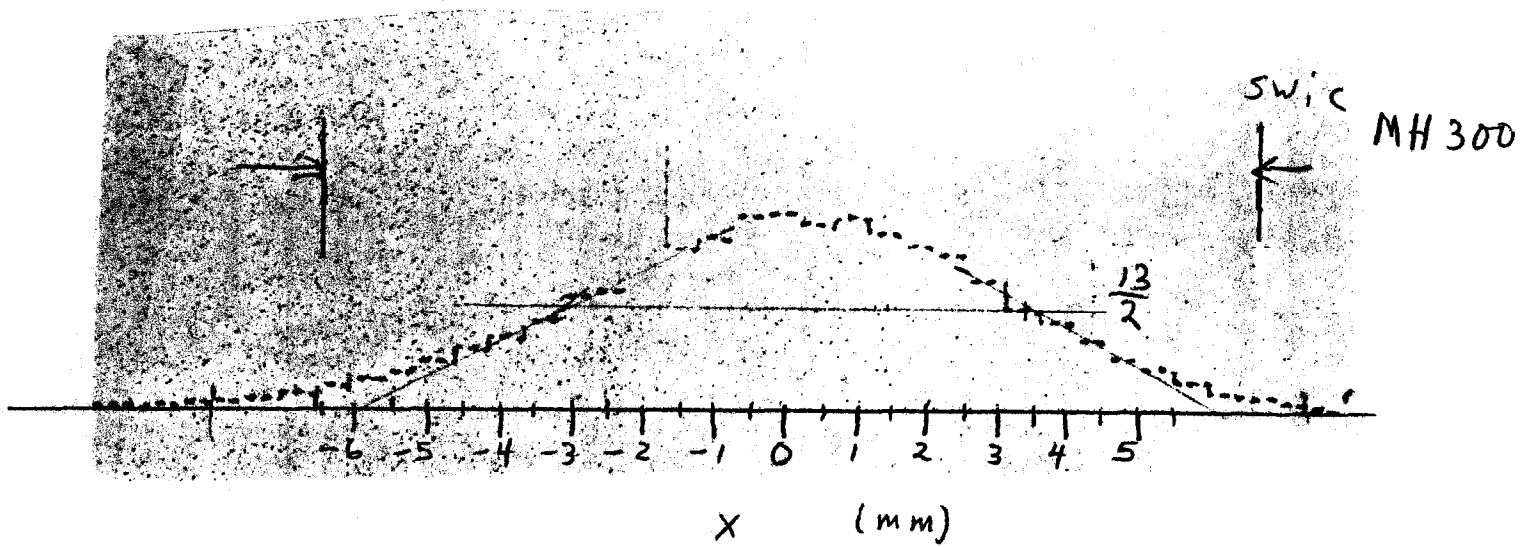
Data:
July 73

swic AO

X (mm)

swic VH

?

X (mm)

swic MQ90

7.9

X (mm)
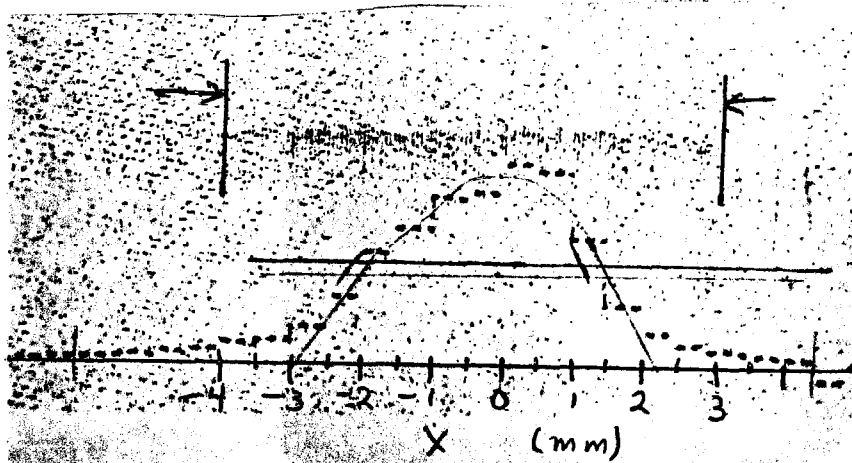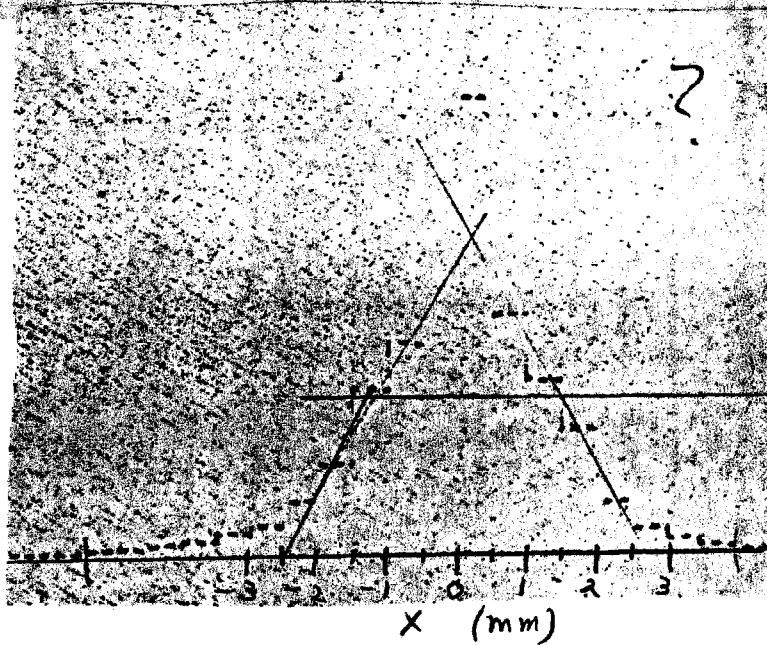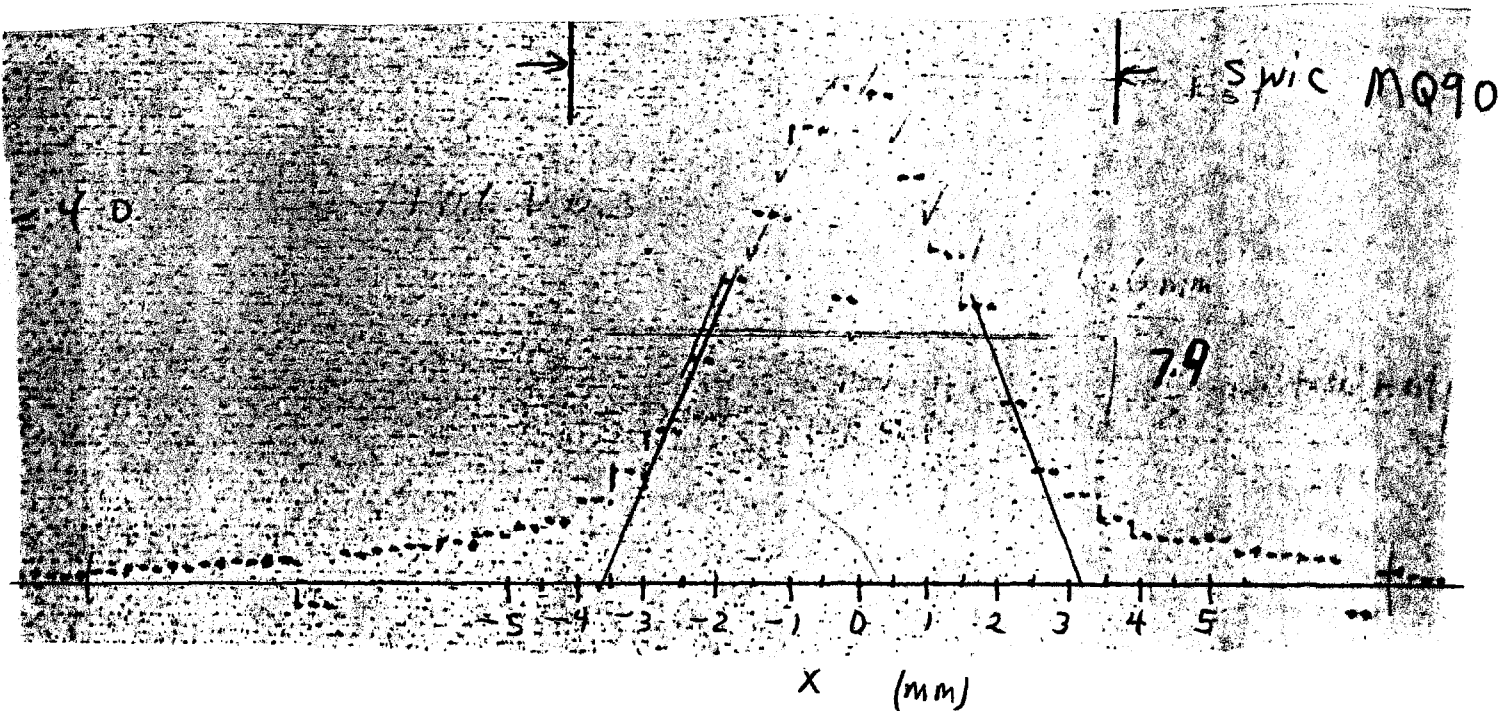
Fig. 4

FWHM of the projections of half-ellipse of area 0.044 π mm-mrad as function of distance from A0

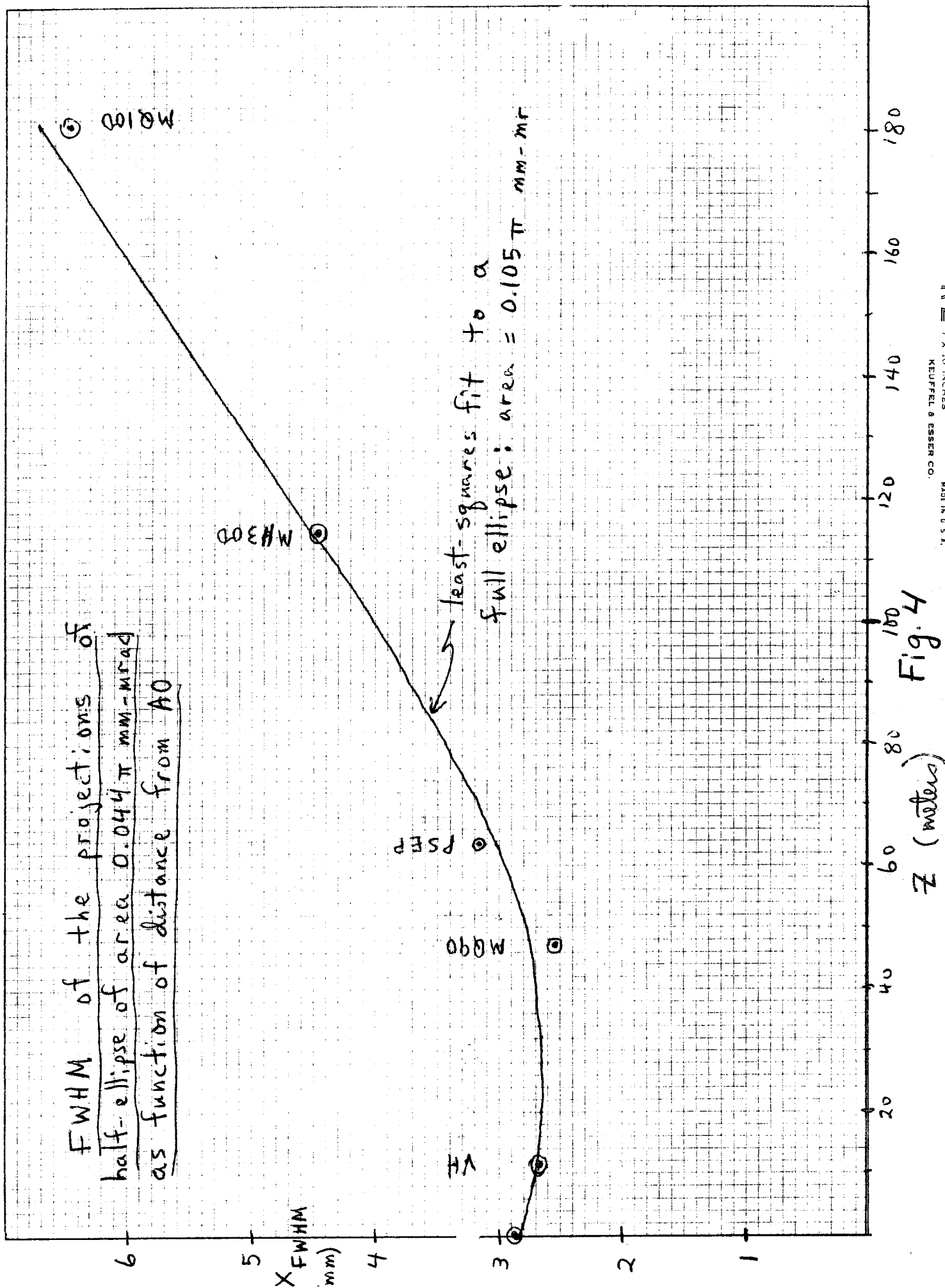least-squares fit to a full ellipse: area = 0.105 π mm-mr

X_{FWHM} (mm)

Z (meters)